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# $\alpha$ -Canonical Form Representation of the Open Loop Dynamics of the Space Shuttle Main Engine

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# $\alpha$ -CANONICAL FORM REPRESENTATION OF THE OPEN LOOP DYNAMICS OF

## THE SPACE SHUTTLE MAIN ENGINE

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### SUMMARY

A parameter and structure estimation technique for multivariable systems is used to obtain a state space representation of open loop dynamics of the space shuttle main engine in  $\alpha$ -canonical form. The parametrization being used is both minimal and unique. The simplified linear models may be used for fault detection studies and control system design and development.

### INTRODUCTION

Accurate representation of the dynamic behavior of the space shuttle main engine (SSME) is required for a variety of diagnostic, control and evaluation purposes. A complete nonlinear dynamic simulation has been developed (Rocketdyne Division of Rockwell International Corporation, 1981). However, its size and complexity make it very difficult to use for design purposes. In a previous study, Duyar, Guo and Merrill, 1990, used a single-input single-output (SISO) identification technique to obtain a transfer function representation of the dynamic behavior of the SSME at 100 percent power level. Although SISO identification approach is quite simple, it results in a nonminimal representation of the SSME. This paper extends the previous work by developing linearized dynamic models of the SSME at five different power levels. A multivariable identification and a minimal realization technique (Eldem and Yildizbayrak, 1988; Eldem and Duyar, 1989) is used to present these models in  $\alpha$ -canonical form. The paper also develops an identification technique for piecewise linear systems with static nonlinear gains, which is used in modeling the dynamics of the SSME.

The identified linearized models are valid in a limited response region about the operating points corresponding to the power levels. The models are useful for real time estimation and fault detection as well as open loop engine dynamic studies and closed loop control analysis using a user generated control law.

Initially a brief description of the SSME is given. This is followed by a description of the identification scheme and the model used. Finally, results obtained from the identified models are compared with the results obtained from the nonlinear simulation for the same input.



## NOMENCLATURE

A,B,C,	state space matrices
k	discrete time integer
MR	oxygen to fuel mixture ratio of the main combustion chamber
N	number of samples
P	pressure
S	speed
T	temperature
t	time
$t_c$	clock time

### Greek Symbols:

$\beta$	valve actuator output rotary motion
$\delta u$	deviation of input from nominal
$\delta x$	deviation of state from nominal
$\delta y$	deviation of output from nominal
$\lambda$	gain

### Subscripts:

C	chamber
CCV	coolant control valve
FPOV	fuel preburner oxidizer valve
HPFT	high pressure fuel turbine
HPOT	high pressure oxidizer turbine
MFV	main fuel valve
MOV	main oxidizer valve
OPOV	oxidizer preburner oxidizer valve

## THE SPACE SHUTTLE MAIN ENGINE

Different aspects of the SSME as well as its principles of operation are described in the literature (Duyar, Guo and Merrill, 1990; Landauer, 1988; Klatt et al., 1982). For the sake of completeness a brief description of the main engine is also given below.

The space shuttle orbiter main propulsion system is composed of three main engines. The engines use liquid oxygen and liquid hydrogen propellants carried in an external tank attached to the orbiter. To understand the overall flow of fuel and oxidizer to produce the thrust, a schematic diagram of the propellant flows and the control valves is shown in figure 1.

The two high pressure turbines are driven by a fuel turbine preburner and an oxidizer turbine preburner, each of which produces hot gas. The low pressure

turbines are driven by the high pressure pump flows. The fuel from the high pressure fuel pump (HPFP) goes through the main fuel valve (MFV). After the MFV, the flow divides into fixed nozzle cooling flow, main chamber cooling flow, and coolant control valve (CCV) flow. Heat is absorbed from the combustion chamber and nozzle. The fixed nozzle cooling and the coolant control valve flows then recombine and travel to the preburners where combustion and pressure are controlled by the fuel preburner oxidizer valve (FPOV) and the oxidizer preburner oxidizer valve (OPOV). Main chamber coolant flow is used to drive the low pressure fuel turbopump.

The existing control system of the SSME uses five valves; FPOV, OPOV, MFV, MOV, and CCV. They control the mixture ratio and the main chamber pressure which is correlated with the thrust. Open and closed loop control of these valves are used to accomplish the SSME mission. A typical SSME mission is shown in figure 2. For the purpose of model identification of the open loop system, the rotary motion of the five valves ( $\beta_{OPOV}$ ,  $\beta_{FPOV}$ ,  $\beta_{CCV}$ ,  $\beta_{MOV}$ ,  $\beta_{MFV}$ ) is used as input. The outputs are the chamber pressure,  $P_c$ , and the mixture ratio, MR.

In a previous study, Duyar, Guo and Merrill, 1990, showed that  $\beta_{CCV}$ ,  $\beta_{MOV}$ , and  $\beta_{MFV}$  valve rotary motions are essentially decoupled from the outputs during the main stage of operation. In this study, only the valve rotary motions,  $\beta_{FPOV}$  and  $\beta_{OPOV}$ , are used as the inputs of the open loop system.

#### SYSTEM IDENTIFICATION

According to the classical definition given by Zadeh, 1962, identification is the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent. This definition is quite general and allows many degrees of freedom in the practical formulation of the identification problem. For example, one may select a linear or a nonlinear model. It is possible to have different parametrizations for a given model. Additional choices need to be made to select the input signal. In this section a comparative review of the literature is given to make these choices.

There are two major classes of mathematical models which are used for identification: linear and nonlinear models. Due to the complexity inherent in nonlinear systems and due to the fact that there is no well established general theory for such systems, it is assumed in this study that the physical process being considered behaves linearly at least within a small neighborhood of a nominal operating point.

Parametrization is an important issue for multi-input multi-output (MIMO) systems since they admit more than one parametrization depending on their observability indices. Thus, in order to obtain a minimal parametrization for MIMO systems, the structure of the system i.e., the observability indices related to each output must be determined. The detailed analysis of structure identification can be found in the literature (Guidorzi, 1975, 1981; Wertz et al., 1982; Overbeek and Ljung, 1982; Beghelli and Guidorzi, 1983; and Correa and Glover, 1984). Recently, Eldem and Yildizbayrak, 1988, employed the notion of output injection to obtain a new canonical form for a special class of observable systems. Through this canonical form, they developed a structure and parameter identification algorithm for a restricted class of systems. Eldem and Duyar, 1989, extended this technique to the class of all minimal systems and constrained the output injection by an extra condition to guarantee the uniqueness of the parametrization. In this study, the



techniques developed by Eldem and Duyar, 1989, are adopted for the identification of the space shuttle main engine dynamics.

Selection of an appropriate input signal is an important step in identification problems. A basic criterion for this selection is that the input/output data should be informative enough to discriminate between different models among the class of models being considered (Ljung, 1987). Without this discrimination there is no guarantee that the obtained parameters are the true parameters of the system. This criterion can be expressed mathematically in terms of the covariance matrices of the input signals and the order of the system being identified (Ljung, 1987; Bitmead, 1984; and Anderson and Johnson, 1982). Uncorrelated pseudo random binary sequences which are used in this study are examples of such input signals which can be used for designing an informative experiment.

Based on the above review, the steps in identification consist of:

(1) selection of a driving signal with persistent excitations, (2) selection of a model, (3) parameter and structure estimation, and (4) model verification. These steps are followed for the identification of the SSME and are outlined below.

#### Selection of a Driving Signal

Pseudo random binary sequences (PRBS) are selected as the input perturbation signals to excite the system because of their convenience and suitability for similar applications (Cottingham and Pease, 1978; Sudhakar et al., 1988). Two uncorrelated sequences, which satisfy the requirement of persistent excitation, are used for OPOV and FPOV inputs. The sequences have a clock time of 0.04 sec and a length of 127. This corresponds to a maximum frequency of 78.5 rad/sec (12.5 Hz), a minimum frequency of 1.24 rad/sec (0.2 Hz) and a signal duration of 5.08 sec.

#### Selection of a Mathematical Model

In a previous study by Duyar, Guo and Merrill, 1990, the existence of significant backlash and stiction nonlinearities associated with the valve dynamics were observed. These two nonlinearities can be isolated from the rest of the system and dealt with separately. Here, the open loop system includes actuator dynamics, the nonlinear element which contains valve stiction and valve linkage backlash, and the engine system which includes all other nonlinearities and the engine dynamics. To simplify the analysis, the backlash and stiction are temporarily removed from the analysis. Now the open loop system, without the backlash and the stiction, is linearized and identified. The backlash and the stiction nonlinearities can later be added to the identified system. In practice values for backlash and stiction are determined from manufacturer's specifications or bench testing of the individual components.

The nonlinear dynamics, except the backlash and the stiction, of the SSME can be described by the nonlinear equations:

$$\dot{x}(t) = f[x(t), u(t)] \quad (1)$$

where  $x$ ,  $u$ , and  $y$  are the state, the control and the output vectors, respectively. Linearizing these equations about a nominal operating condition and discretizing

$$y(t) = g[x(t)] \quad (2)$$

yields:

$$\delta x(k+1) = A \delta x(k) + B \delta u(k) \quad (3)$$

$$\delta y(k) = C \delta x(k) \quad (4)$$

where  $\delta x$ ,  $\delta u$ , and  $\delta y$  are the deviations of the state, the input, and the output vectors about the nominal operating condition. It is assumed that the system described by equations (3) and (4) is stable and observable and the  $C$  matrix has full row rank. Furthermore, this system is realized in  $\alpha$ -canonical form (Eldem and Yildizbayrak, 1988; and Eldem and Duyar, 1989), i.e., the following relations hold:

$$C = [0 : H^{-1}] \quad (5)$$

$$A = A_o + KHC, \text{ with } A_o^{\mu} = 0 \quad (6)$$

$$(HC)_{r_i} A_o^{\mu_i} = 0 \quad (7)$$

$$(HC)_{r_i} A_o^k K_{c_j} = 0, \text{ for } \mu_i > \mu_j \text{ and } k < \mu_i - \mu_j \quad (8)$$

Here the subscripts  $r_i$  and  $c_j$  denote the  $i$ 'th row and  $j$ 'th column respectively. Superscripts indicate exponentiation. The structure matrix  $A_o$  is lower left triangular and consists of zeros and ones only and is determined by the observability indices,  $\mu_i$  where  $i$  associates  $\mu_i$  with the  $i$ 'th output and  $\mu = \max \{\mu_i\}$ . The matrix  $K$  is a deadbeat observer gain.

In the following section a parameter and structure identification scheme for the above system is given. It is assumed that  $H$  matrix is equal the identity matrix,  $I$ . The generalization of this identification scheme to the case with  $H \neq I$  is treated by Eldem and Duyar, 1989.

#### Identification of Linear Models

Throughout this section a linear, discrete time, multivariable system described by equations (3) and (4) is considered. The triplet  $(C, A, B)$  satisfies equations (5) to (8) with  $H = I$ . The expression for the state  $\delta x(k)$  at time  $k$  of this system is given as:

$$\delta x(k) = A_o^k \delta x(0) + \sum_{i=1}^{\mu} A_o^{i-1} [K B] \begin{bmatrix} \delta y(k-i) \\ \delta u(k-i) \end{bmatrix} \quad (9)$$

Using the nilpotency of  $A_o$  the above equation yields:

$$\delta x(k) = \sum_{i=1}^{\mu} A_o^{i-1} [K B] \begin{bmatrix} \delta y(k-i) \\ \delta u(k-i) \end{bmatrix}, \quad \text{for } k \geq \mu \quad (10)$$

This implies that

$$\delta y(k) = \sum_{i=1}^{\mu} C A_o^{i-1} [K B] \begin{bmatrix} \delta y(k-i) \\ \delta u(k-i) \end{bmatrix}, \quad \text{for } k \geq \mu \quad (11)$$

Now let

$$\theta_y = \begin{bmatrix} CK & C A_o K & \dots & C A_o^{\mu-1} K \end{bmatrix} \quad (12)$$

$$\theta_u = \begin{bmatrix} CB & C A_o B & \dots & C A_o^{\mu-1} B \end{bmatrix} \quad (13)$$

$$\phi_y = \begin{bmatrix} \delta y(k-1)^T & \dots & \delta y(k-\mu)^T \end{bmatrix}^T \quad (14)$$

$$\phi_u = \begin{bmatrix} \delta u(k-1)^T & \dots & \delta u(k-\mu)^T \end{bmatrix}^T \quad (15)$$

and

$$\theta = \begin{bmatrix} \theta_y & \theta_u \end{bmatrix} \quad (16)$$

$$\phi(k) = \begin{bmatrix} \phi_y(k)^T & \phi_u(k)^T \end{bmatrix}^T \quad (17)$$

Using the notation above, equation (11) can be written as:

$$\delta y(k) = \theta \phi(k), \quad \text{for } k \geq \mu \quad (18)$$

If the length of the input/output data is  $N$ , then:

$$Y_N = [\delta y(\mu) \delta y(\mu+1) \dots \delta y(N)] \quad (19)$$

$$Y_N = \theta \begin{bmatrix} \phi_Y(\mu) & \dots & \phi_Y(N) \\ \phi_u(\mu) & \dots & \phi_u(N) \end{bmatrix} \quad (20)$$

Defining  $Q$  and  $R$  as

$$Q = [\phi_Y(\mu) \dots \phi_Y(N)] \quad (21)$$

$$R = [\phi_u(\mu) \dots \phi_u(N)] \quad (22)$$

we obtain

$$Y_N = \theta_Y Q + \theta_u R \quad (23)$$

and postmultiplying the above equation with  $[Q^T \ R^T]$  yields:

$$\bar{Y}_N = Y_N [Q^T \ R^T] \quad (24)$$

$$\bar{Y}_N = \begin{bmatrix} \theta_Y & \theta_u \end{bmatrix} \begin{bmatrix} QQ^T & QR^T \\ RQ^T & RR^T \end{bmatrix} \quad (25)$$

Defining  $\Phi$  as:

$$\Phi = \begin{bmatrix} QQ^T & QR^T \\ RQ^T & RR^T \end{bmatrix} \quad (26)$$

Equation (25) can be rewritten as:

$$\bar{Y}_N = \theta \Phi \quad (27)$$

Equation (27) can be solved to determine the parameters  $\theta_Y$  and  $\theta_u$  if the observability indices  $\mu_i$  are known. To determine them, an expression for the observability matrix is derived. Toward this end first a matrix  $Q_o$  is defined as:

$$Q_o = QQ^T - QR^T(RR^T)^{-1}RQ^T \quad (28)$$

Under the assumption that inputs are "persistently exciting"  $RR^T$  is nonsingular. Also let  $V^\mu$  and  $U_{N-\mu+1}^\mu$  be the observability and controllability matrices written for  $\mu$  and  $N-\mu+1$  steps respectively, i.e.,



$$V_{\mu} = [CA^{\mu-1} \ CA^{\mu-2} \ \dots \ C]^T \quad (29)$$

$$U_{N-\mu+1} = [B \ AB \ \dots \ A^{N-\mu}B] \quad (30)$$

Then it has been shown by Eldem and Duyar (1989) that the following expression holds:

$$Q_o = V_{\mu} M V_{\mu}^T \quad (31)$$

Here  $M$  is a nonsingular matrix defined as:

$$M = U_{N-\mu+1} R_o \left[ I - R^T (R R^T)^{-1} R \right] R_o^T U_{N-\mu+1}^T \quad (32)$$

and  $R_o$  is given below:

$$R_o = \begin{bmatrix} \delta u(0) & \delta u(1) & \dots & \delta u(N - \mu) \\ 0 & \delta u(0) & \dots & \delta u(N - \mu - 1) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \delta u(0) \end{bmatrix} \quad (33)$$

Equation (31) has an important consequence: row by row rank search on  $Q_o$  yields the observability indices of the system. This is due to the fact that post-multiplying the observability matrix  $V_{\mu}$  by  $M V_{\mu}^T$  does not change the observability indices since  $M$  is nonsingular and  $V_{\mu}^T$  has full row rank. Thus equation (27) can now be solved for the parameter matrix  ${}^{\mu}\theta$ , using the least square technique.

#### Identification of Piecewise Linear Models

The procedure outlined earlier is used to identify the open loop system dynamics of the SSME at different power levels. The responses of the identified open loop system are compared with the responses obtained from the nonlinear simulation. In general, it is observed that the gains for positive and negative perturbation signals are significantly different. In order to compensate for this phenomenon, a system gain with different values for positive and negative perturbations is added to the linearized model.

With this modification the system equations become

$$\delta x(k+1) = A \delta x(k) + B \delta u^*(k) \quad (34)$$

$$\delta y(k) = C \delta x(k) \quad (35)$$

where  $\delta u^*(k)$  is defined component-wise as

$$\delta u_i^* = \begin{cases} \lambda_i \delta u_i(k) , & \text{for } \delta u_i(k) \geq 0 \\ (2 - \lambda_i) \delta u_i(k) , & \text{for } \delta u_i(k) \leq 0 \end{cases} \quad (36)$$

The input sequence  $\delta u(k)$  can now be decomposed as

$$\delta u(k) = \delta u^+(k) + \delta u^-(k) \quad (37)$$

where

$$\delta u_i^+(k) = \begin{cases} \delta u_i(k) , & \text{for } \delta u_i(k) > 0 \\ 0 , & \text{otherwise} \end{cases} \quad (38)$$

$$\delta u_i^-(k) = \begin{cases} \delta u_i(k) , & \text{for } \delta u_i(k) < 0 \\ 0 , & \text{otherwise} \end{cases} \quad (39)$$

Using the above definitions the system equations can be written as

$$\begin{aligned} \delta x(k+1) &= A \delta x(k) + B \delta u(k) + B(\Lambda - I) [\delta u^+(k) - \delta u^-(k)] \\ \delta y(k) &= C \delta x(k) \end{aligned} \quad (40)$$

where  $\Lambda = \text{diag } \{\lambda_i\}$ . Using  $\alpha$ -canonical form ( $A = A_0 + KC$ ) and the properties of  $A_0$  an expression for  $\delta y(k)$  can be written as

$$\delta y(k) = \theta_y \phi_y(k) + \theta_u \phi_u(k) + \theta_u (\Lambda - I) [\phi_u^+(k) - \phi_u^-(k)] \quad (41)$$

For constant amplitude perturbations (for instance PRBS) the last term in the above equation is constant since

$$\delta u_i^+(k) - \delta u_i^-(k) = |\delta u_i(k)|$$

Thus, taking the ensemble averages in equation (41) yields

$$\delta y_m = \bar{\theta}_y \delta y_m + \bar{\theta}_u \delta u_m + \bar{\theta}_u (\Lambda - I) \gamma e \quad (42)$$

where  $\delta y_m$  and  $\delta u_m$  denote the average values of  $u(\cdot)$  and  $y(\cdot)$ , respectively. Furthermore,

$$\bar{\theta}_y = \sum_{i=1}^{\mu} C A_o^{i-1} K H \quad (43)$$

$$\bar{\theta}_u = \sum_{i=1}^{\mu} C A_o^{i-1} B \quad (44)$$

$e$  is an  $m$ -vector consisting of ones and  $\gamma$  is the magnitude of the perturbations. Solving equation (42) for  $\lambda_i$ 's we obtain

$$\lambda = \frac{1}{\gamma} \left( \bar{\theta}_u^T \bar{\theta}_u \right)^{-1} \bar{\theta}_u^T \left[ (I - \bar{\theta}_y) \delta y_m - \bar{\theta}_u \delta u_m \right] e \quad (45)$$

where  $\lambda = [\lambda_1 \lambda_2 \dots \lambda_m]^T$ ,  $e = [1 \ 1 \dots 1]^T$ .

Note that using equation (42) and the fact that  $u^+(k) - u^-(k) = \text{constant}$ , the piecewise linear system defined by equations (34) to (36) can be transformed into a purely linear system as follows:

$$\text{Let } \tilde{x}(k) = \delta x(k) - \delta x_m$$

$$\tilde{y}(k) = \delta y(k) - \delta y_m$$

$$\tilde{u}(k) = \delta u(k) - \delta u_m$$

Then, by equations (40) and (42), it follows that

$$\tilde{x}(k+1) = A \tilde{x}(k) + B \tilde{u}(k) \quad (46)$$

$$\tilde{y}(k) = C \tilde{x}(k) \quad (47)$$

Now the identification procedure outlined in this section can be used to determine  $(C, A, B)$  of the above system. Since  $C, A, B$  is assumed to be in  $\alpha$ -canonical form, i.e.,  $A = A_o + KC$ ,  $\bar{\theta}_y$  and  $\bar{\theta}_u$  can be calculated by equations (43) and (44). Then, using the ensemble averages  $\delta y_m$  and  $\delta u_m$  in equation (45) we can obtain the system gains  $\{\lambda_i\}$ .

## RESULTS

Using the above procedure and the identification algorithm presented in this paper, the parameters of the piecewise linear model described by equations (34) to (36) are identified at 70, 80, 90, 100, and 110 percent power levels. As



mentioned earlier, the input vector,  $\delta u$ , represents the deviations of the valve rotary motion from the nominal values defined as:

$$\delta u = [\delta\beta_{\text{OPOV}} \quad \delta\beta_{\text{FPOV}}]^T \quad (48)$$

The output vector  $\delta y$  represents the deviations of the chamber inlet pressure and the mixture ratio from their nominal values and defined as:

$$\delta y = [\delta P_c \quad \delta MR]^T \quad (49)$$

The A,B,C matrices, the nonlinear gains  $\lambda_1$  and  $\lambda_2$  and the observability indices of the identified piecewise linear model are given in table I.

The validity of the estimated parameters of the system is checked by comparing the responses obtained from the identified system with the response of the nonlinear simulation. Both a state variable filter and the model of the identified system are used for comparison purposes. The state variable filter utilizes the measurements of both the output  $\delta y$ , and the input,  $\delta u$ , to estimate the next value of the output,  $\delta y_f(k)$ , defined as

$$\delta y_f = \sum_{i=1}^{\mu} CA_o^{i-1} [K \ B] \begin{bmatrix} \delta y(k-i) \\ \delta u^*(k-i) \end{bmatrix} \quad (50)$$

The model of the identified system utilizes only the measurement of the input data,  $\delta u$ , to predict the output,  $\delta y_m$ , and the state,  $\delta \hat{x}$ , as

$$\delta \hat{x}(k+1) = A\delta \hat{x}(k) + B\delta u^*(k) \quad (51)$$

$$\delta y_m(k) = C\delta \hat{x}(k) \quad (52)$$

Two different test input signals are used for comparison purposes. The first test input signal consists of two full length PRBS (different PRBS than the ones used for identification purposes) which cover the same frequency range as those used for identification. The second test signal consists of step and ramp inputs as shown in figure 3, and covers a lower frequency range than the range of validity of the identified system.

In order to compare the responses of the identified system with the responses obtained from the nonlinear simulation, a standard error of estimate (SEE) is defined as

$$SEE = \sqrt{\frac{\sum_{k=1}^N [\delta z_i(k) - \delta y_i(k)]^2}{\sum_{k=1}^N \delta y_i(k)^2}} \quad (53)$$

Here the subscript  $i$  denotes the  $i^{th}$  element of the output vector  $\delta y$  and  $\delta z$  refers either to the model output, i.e.,  $\delta z = \delta y_m$  or the filter output, i.e.,  $\delta z = \delta y_f$ . The SEE's obtained for both of the test signals, given in table II, indicate very good agreement. The comparison of the responses of both the identified model and the filter to the responses obtained from the nonlinear simulation of the SSME for low frequency test signals also indicate good agreement as shown in figures 4 to 8.

#### CONCLUSION

A multivariable system identification technique is used to represent the dynamic behavior of the SSME at five different power levels. Minimal realizations of these models in state space representation using  $\alpha$ -canonical form are given. An identification technique to determine static nonlinear gain changes is also presented.

The comparison of the responses of the nonlinear simulation with the responses of identified models indicates very good agreement and can be used for control design purposes. Since the identified models are valid for limited response regions, this study will be extended so as to obtain models at other operating points. Then these point models will be linked to obtain a simplified model of the SSME covering its full range of operation.

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TABLE I. - IDENTIFIED SYSTEM PARAMETERS AT 70, 80, 90, 100, and 110 PERCENT POWER LEVELS

Power level, percent	A				B		C				$\lambda_1$	$\lambda_2$
70	0	0	-0.2553	0.0416	0.3923	0.0958					0.8957	0.8403
	0	0	0.5044	-0.1108	1.0558	-0.2652	0	0	1	0		
	1	0	1.0218	-0.113	0.2584	0.0788	0	0	0	1		
	0	1	-0.7902	0.5812	0.5902	-0.2697						
80	0	0	-0.1576	0.0257	0.4068	0.1564					1.1211	0.9279
	0	0	0.2587	-0.0545	0.9631	-0.674	0	0	1	0		
	1	0	0.8759	-0.0666	0.2574	0.0384	0	0	0	1		
	0	1	-0.5157	0.4766	0.6046	-0.2307						
90	0	0	-0.1206	0.0155	0.4223	0.1546					0.9386	0.7699
	0	0	0.1434	-0.0275	0.9217	-0.7302	0	0	1	0		
	1	0	0.8117	-0.0478	0.2518	0.021	0	0	0	1		
	0	1	-0.4183	0.4226	0.6288	-0.1779						
100	0	0	-0.1279	0.0212	0.3534	0.0475					0.8594	0.9047
	0	0	0.1994	-0.0508	0.6831	-0.6446	0	0	1	0		
	1	0	0.7816	-0.0565	0.2007	-0.0181	0	0	0	1		
	0	1	-0.5253	0.475	0.5043	-0.1387						
110	0	0	-0.084	0.0026	0.2416	0.0786					0.9704	1.1231
	0	0	0.1082	-0.0123	0.5112	-0.575	0	0	1	0		
	1	0	0.728	0.0181	0.1456	-0.0243	0	0	0	1		
	0	1	-0.4164	0.333	0.3593	-0.078						

TABLE II. - STANDARD ERROR OF ESTIMATES (SEE) OF PREDICTIONS OF THE FILTER AND THE MODEL

Operating condition					
	PRBS signal				
	Power, percent				
	70	80	90	100	110
Filter					
$P_c$	0.0648	0.0488	0.0443	0.0441	0.0985
MR	0.0876	0.1028	0.088	0.0526	0.0822
Model					
$P_c$	0.0793	0.0951	0.0819	0.0781	0.124
MR	0.0667	0.0943	0.0816	0.0538	0.0903
	Low frequency test signal				
	Power, percent				
	70	80	90	100	110
Filter					
$P_c$	0.0123	0.0422	0.0508	0.0356	0.0204
MR	0.0483	0.0721	0.1024	0.074	0.1049
Model					
$P_c$	0.0481	0.14	0.1536	0.0926	0.0469
MR	0.0911	0.0858	0.1337	0.1002	0.1639

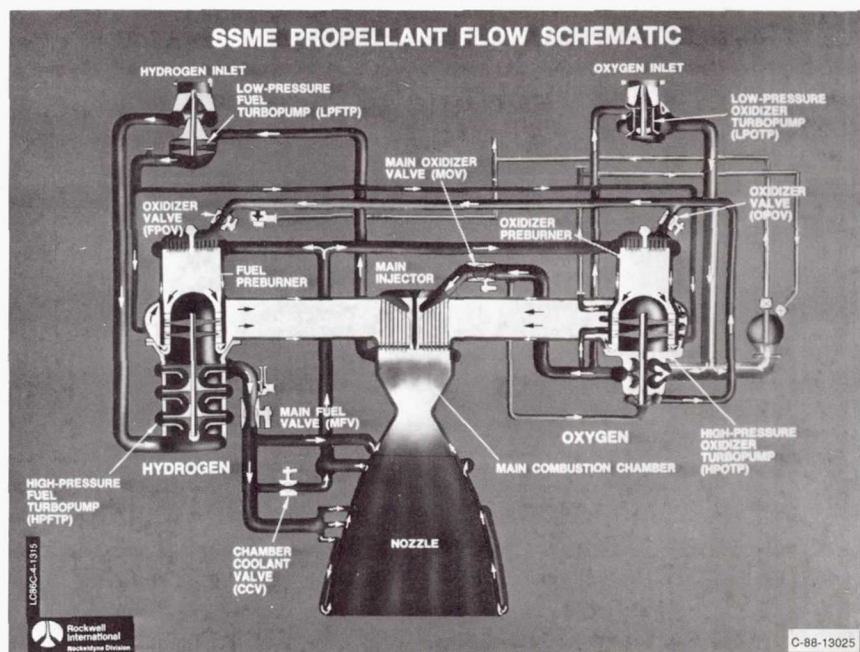


Figure 1.—Propellant flow and control valves (Rockwell, 1988).

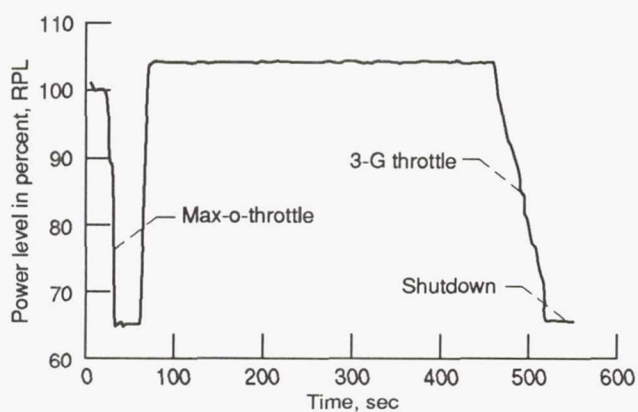


Figure 2.—Typical 104% SSME mission (Rockwell, 1988).

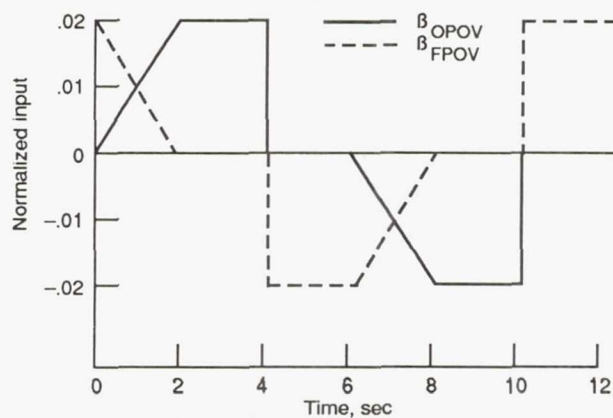


Figure 3.—Low frequency test input signals,  $\beta_{OPOV}$  and  $\beta_{FPOV}$ .

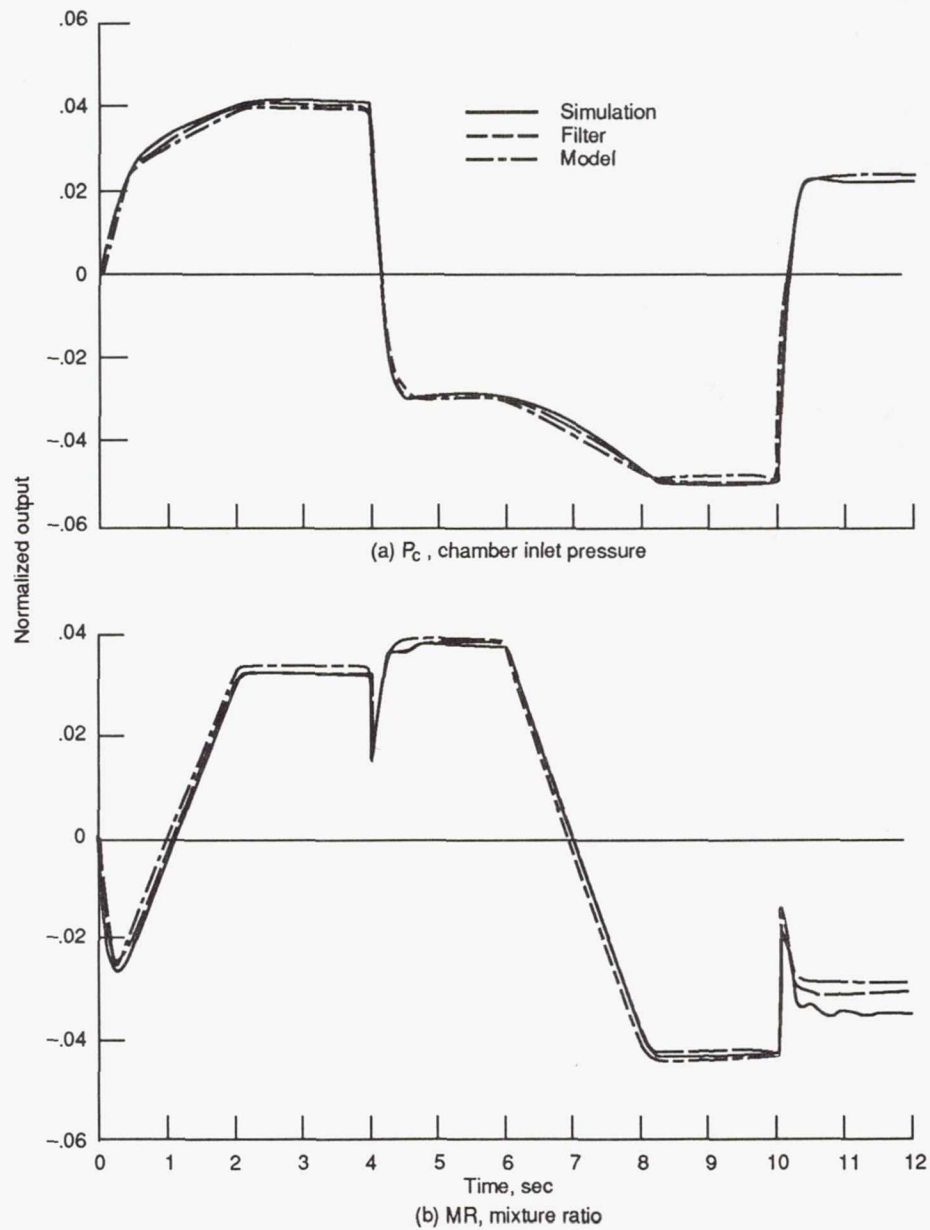


Figure 4.—Comparison of the responses of the identified model and the state variable filter with the nonlinear simulation at a 70% power level.



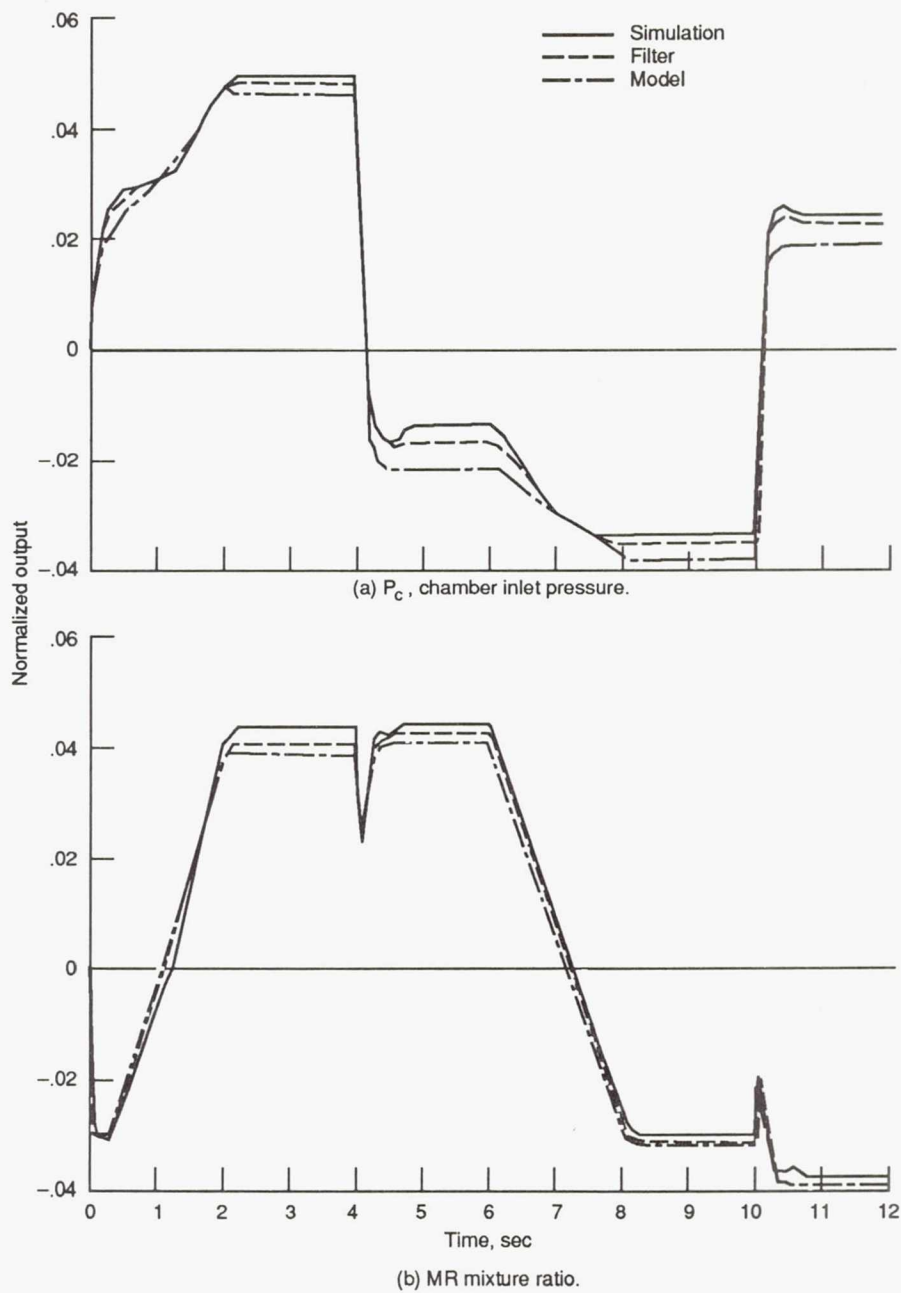


Figure 5.—Comparison of the responses of the identified model and the state variable filter with the nonlinear simulation at an 80% power level.

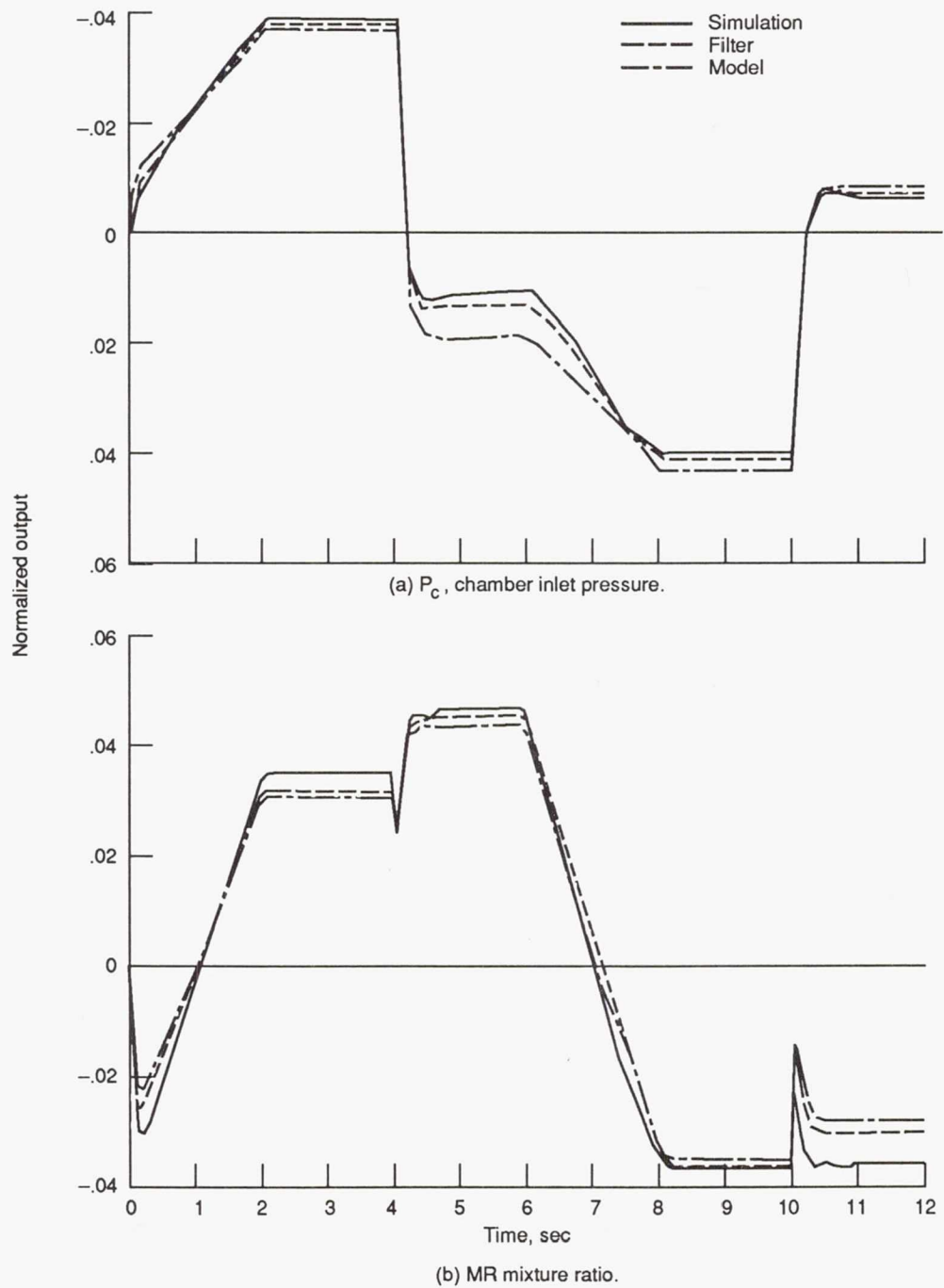


Figure 6.—Comparison of the responses of the identified model and the state variable filter with the nonlinear simulation at a 90% power level.

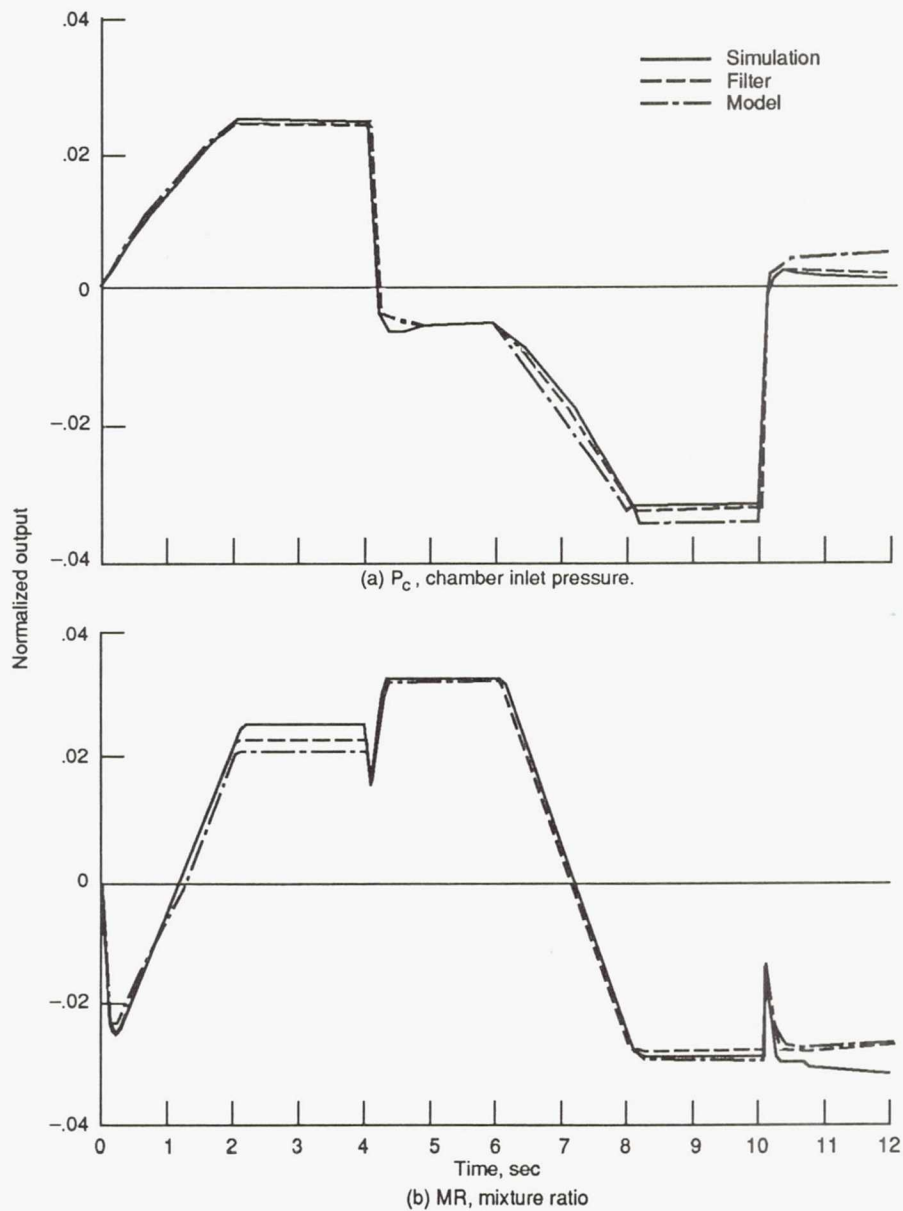


Figure 7.—Comparison of the responses of the identified model and the state variable filter with the nonlinear simulation at 100% power level.



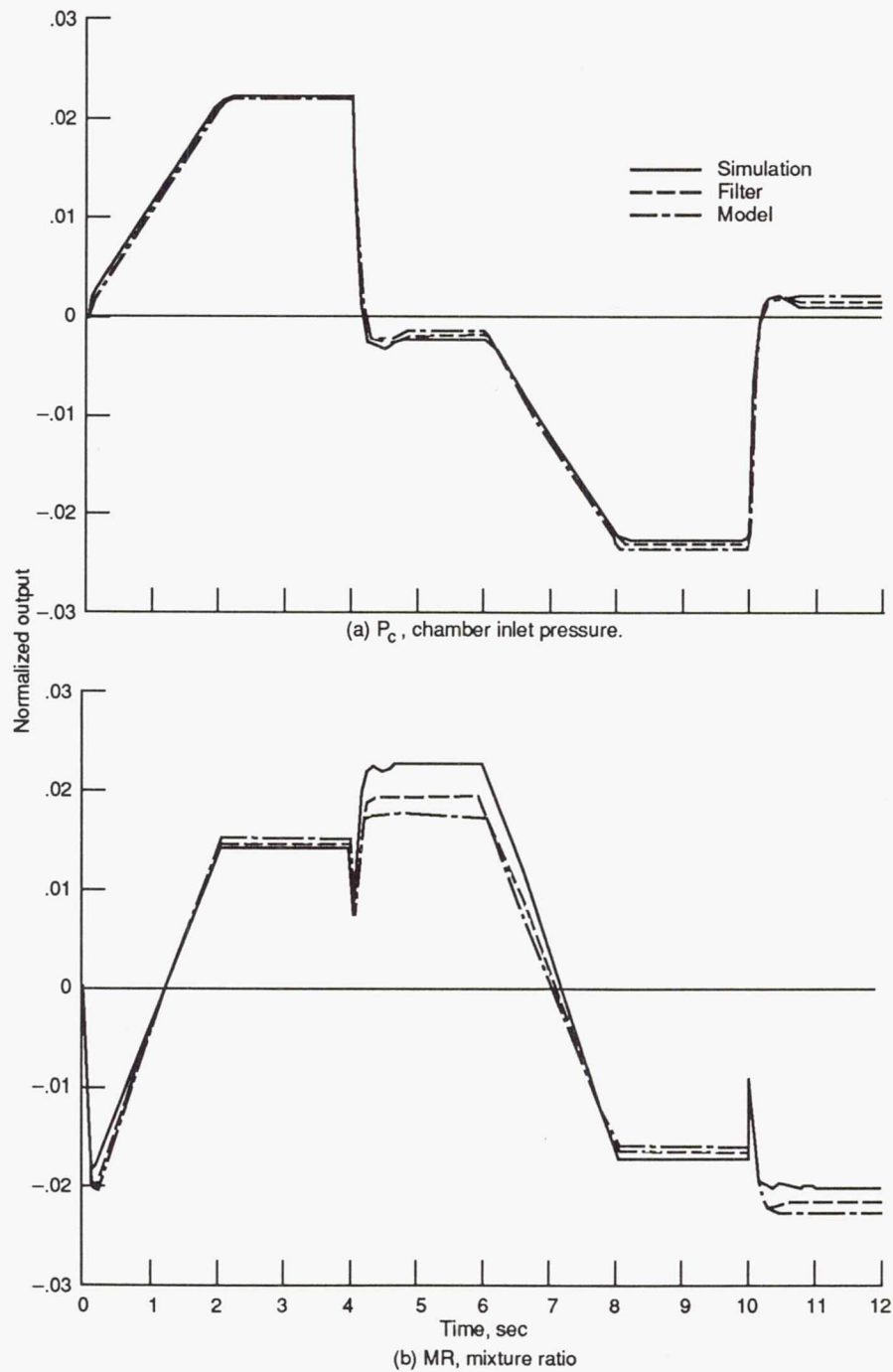


Figure 8.—Comparison of the responses of the identified model and the state variable filter with the nonlinear simulation a 110% power level.



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## Report Documentation Page

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16. Abstract A parameter and structure estimation technique for multivariable systems is used to obtain a state space representation of open loop dynamics of the space shuttle main engine in $\alpha$ -canonical form. The parametrization being used is both minimal and unique. The simplified linear models may be used for fault detection studies and control system design and development.			
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